be a far better introduction to nomography than an analytical approach by the theory of determinants. Basic analytic theory is not neglected, however, and a chapter on determinants is included near the end.

The second edition is considered a definite improvement over the first; it has a more pleasing format and type arrangement and an over-all attractiveness that gives it more pedagogic appeal. (One minor defect: chapter numbers on the top of each page of the first edition were unaccountably omitted in the second edition.)

Besides numerous text revisions, important new material appears in the second edition. Most significant are: (a) the expansion of the chapter on determinants, (b) the addition of a chapter on projective transformations, and (c) the addition of a chapter indicating the relationship between concurrency and alignment nomograms (with applications to experimental data, including a description of the rectification of experimental curves).

An appendix supplies an assortment of nomographic solutions to different problems taken from various technical fields, and offers fine illustration of available techniques.

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39[X, Z].—LYLE R. LANGDON, *Approximating Functions for Digital Computers*. Reprinted from *Industrial Mathematics* v. 6, 1955, p. 79–100.

This article is concerned with several methods for determining approximations to functions of one real variable. The methods mentioned include the use of Padé approximants, a modification of the Taylor expansion said to be due to Obrechkoff, the use of Chebyshev polynomials to economize truncated power series in the sense of Lanczos, the use of a rational interpolant which collocates $f(x)$ at five points, as well as some special devices based on a study of the particular function to be approximated.

The following approximations are given:

Function	Nature of approximation	Range of x	Stated upper bound for error
$\sin x$	Rational	$-\pi \leqq x \leqq \pi$	6×10^{-9}
$\cos x$	Rational	$-\pi \leqq x \leqq \pi$	1×10^{-9}
$\tan x$	Rational	$-\pi/4 \leqq x \leqq \pi/4$	7×10^{-8}
e^x	Rational	$-\pi \leqq x \leqq \pi$	1 unit in signific digit
$\sqrt{x}$	Rational	$0.1 \leqq x \leqq 10.0$	Something in 5th significant digit
$\sin \frac{\pi x}{2}$	Polynomial	$-1 \leqq x \leqq 1$	4×10^{-9}
$\cos \frac{\pi x}{2}$	Polynomial	$-1 \leqq x \leqq 1$	7×10^{-9}

Function	Nature of approximation	Range of x	Stated upper bound for error
$\log_e x$	Polynomial in $y = \frac{x-a}{x+a}$, a being a suitably chosen constant	$0.10 \leqq x \leqq 100.0$ (for various choices of a)	3×10^{-9}
$\tan^{-1} y$	Polynomial	$-(\sqrt{2}-1) \leqq y \leqq (\sqrt{2}-1)$	4×10^{-9}
(used to represent $\tan^{-1}x$ on $0 \leqq x \leqq 1$ and $1 \leqq x \leqq \infty$ by expressing y as the ratio of two appropriately chosen linear functions of x)			
$\sin^{-1} x$	Polynomial	$-\frac{1}{2} \leqq x \leqq \frac{1}{2}$	3×10^{-9}
$\sin^{-1} x$	$\frac{\pi}{2} - \sqrt{1-x}\cdot$polyn. in x	$0.966 \leqq x \leqq 1.0$	3×10^{-9}
$H(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$	Polynomial	$0 \leqq x \leqq 1$	6×10^{-10}
	Polynomial	$1 \leqq x \leqq 2$	2×10^{-10}
	Polynomial	$2 \leqq x \leqq 3$	8×10^{-9}
	Polynomial	$3 \leqq x \leqq 4$	7×10^{-9}
$\cosh x \cos x$	Polynomial	$-\frac{\pi}{2} \leqq x \leqq \frac{\pi}{2}$	1×10^{-10}
$\cosh x \sin x + \sinh x \cos x$	Polynomial	$-\frac{\pi}{2} \leqq x \leqq \frac{\pi}{2}$	2×10^{-10}
$\sinh x \sin x$	Polynomial	$-\frac{\pi}{2} \leqq x \leqq \frac{\pi}{2}$	1×10^{-8}
$\cosh x \sin x - \sinh x \cos x$	Polynomial	$-\frac{\pi}{2} \leqq x \leqq \frac{\pi}{2}$	3×10^{-9}

(Linear combinations of the last four functions give such functions as

$$e^{\pi x/2} \sin \pi x/2, \int e^x \sin xdx, \text{etc.})$$

In addition, approximations of certain functions occurring in *Gas Tables* by J. H. Keenan & Joseph Kaye, Wiley & Sons, 1945, are given.

The paper has been reprinted in both an "uncorrected" and a "corrected" version. The following typographical errors were found to remain in the "corrected" version:

Page		
87	line 4 *fb*	Decimal point is missing from .00832 $86830x^5$, and x^7 is missing from 0.00019 $22123x^7$.
89	lines 15, 17, 18, 23, 25	Approximate value of $\sqrt{2} - 1$ should end in 4 instead of 3.
90	lines 8, 10	(same as above)
93–4	Eqs. (15), (16), (17)	Prefix a_o to the given integral.
93	Eq. (15)	Summation should be $\sum_1^9 a_n(2x-3)^n$.
99	Range of h for Eq. (26)	Should read $395.74189\ 76 < h \leqq 513.4$.

The reviewer compared Langdon's results with those of Cecil Hastings, Jr., *Approximations for Digital Computers*, where possible. Although both authors considered approximations for $\tan^{-1} x$, $\sin (\pi/2)x$, $\sin^{-1} x$, and $H(x)$, the only one of these in which the ranges are the same $(-1 \leqq x \leqq 1)$ is that for $\sin (\pi/2)x$. Here both authors use a five-term polynomial approximant; Langdon claims an error bound of 4×10^{-9}, compared to Hastings' claim of 5×10^{-9}. It is of interest to note that C. W. Clenshaw approximated the same function on the same range with a six-term polynomial, the stated error bound being 3×10^{-9} (*MTAC*, v. 1954, p. 143).

The reviewer made some spot checks of the approximations given and found no evidence that the error bounds or coefficients are incorrect, with the exception that the value of $H(2.5)$ was found to be 0.99959 30388, as compared with the value 0.99959 30480 given in the NBS *Tables of Probability Functions*, v. 1. Here the error in the value obtained from Langdon's approximation exceeds the stated bound of $8 \cdot 10^{-9}$.

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40[Z].—H. FREUDENTHAL, "Logique Mathématique Appliquée," *Collection de Logique Mathématique*, Série A, XIV, Gauthier-Villars, Paris, 1958, 57 p. Price $3.00.

This pamphlet is a prize-winning essay in a 1953 contest of the Institute for the Unity of Sciences on the general subject, Mathematical Logic as a Tool of Analysis—its uses and achievements in the sciences and philosophy. For the purposes of the present publication the most interesting portion is the second section entitled "Le calcul des propositions, les réseaux electriques et les machines à calculer." This gives a brief description of various electronic and electro-mechanical realizations of the propositional calculus, introduces the temporal problem, and suggests research in a temporally affected propositional calculus. Unfortunately, the span of five years between initial composition and publication means that no knowledge of the recent work on such temporal structures as the neuron model is demonstrated. The author's point that mathematical logicians should enter the field rather than surrender it to computer engineers retains its initial force.

Briefly, the introductory section comments on what the author considers as an unfortunate tendency to abstract, non-realizable research by logicians. (He concedes the right of number theorists to an ivory tower but thinks that logic must be primarily viewed as a part of applied mathematics.) The third section reviews certain clarifications introduced in philosophy and the foundations of science by the influence of mathematical logic. The fourth suggests an operational definition of implication in terms of an idea of "complete implication" which requires that all pairs (p, q) such that $p \rightarrow q$ be meaningful in a particular case before using the term "implication." The final portion suggests problems involving transitivity in modal forms.

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